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ON UMOV'S WORKS

(1846–1915)

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NIKOLAI ALEKSEYEVICH UMOV, the famous Russian physicist, was born in Simbirsk. In 1863, he entered Moscow University. In 1867, after graduating from the University he remained to gain professorship. In 1870, he passed master's examinations successfully and within a year submitted a thesis for master's degree. The thesis was devoted to thermomechanical phenomena in solid–elastic bodies in a non-uniform temperature field. In 1871, Umov was selected as a reader and in 1875 as a professor at the Novorossiisk University in Odessa. Here he made friends with I. M. Sechenov and I. I. Mechnikov who played an important part in the formation of his materialistic world outlook. In 1893, Umov returned to Moscow University where, from 1896, after Stoletov's death, he became head of the department of physics. Umov devoted much effort and energy to organizing and building a new physical Institute at Moscow University. Here he worked till 1911 when he, with other progressive scientists left Moscow University in protest against actions of the Minister of Education, L. Kasso. Later on he worked in the Moscow Society of Nature Testers where, from 1897, he was President for seventeen years and in "Ledentsov's Society of Assistance in Achievements of Experimental Sciences and their Practical Applications".

Umov was a remarkable propagandist for science. He wrote a great number of popular scientific articles and delivered many public lectures. Umov took part in organizing various scientific societies (Pedagogical, Ledentsov's Society) and was a member of the Society for Amateurs of Natural Science, Anthropology and Ethnography.

1.

Umov's first work "Laws of Oscillations in an Infinite Medium of Constant Elasticity" was published in *Mathematical Collections* in March, 1870. The problem of oscillation in an infinite medium is given full scope by Umov and developed with great elegance and profundity. The results of this work have yet to be outdated. This work, rich in ideas and investigation methods, may still influence the development of science.

At present one of the essential problems of molecular physics is the creation of a theory on heat phenomena in solid and, particularly, in liquid bodies. The view on the nature of heat is gaining confirmation in science in which heat is considered as ultrasonic oscillations propagating in a chaotic manner within liquid or solid. Umov's work cited will be of undoubted use in confirming and developing these ideas. In spite of a lapse of eighty years the ideas are so fresh and fundamental that this work may undoubtedly take its place with modern works dealing with the nature of heat.

Whilst still a student, Umov with great skill used the Lamé method of curvilinear co-ordinates when considering oscillatory systems in an infinite medium of constant elasticity. Applying such a system of curvilinear co-ordinates to the medium, in which a wave surface represents one of the families of co-ordinate surfaces and taking a portion of a ray as parameter of this family, Umov proves that the corresponding differential parameter of the first order will be equal to unity. Such a choice of co-ordinates allowed him to separate problems of transverse and of longitudinal oscillations in an infinite medium. Thus he succeeded in obtaining a number of interesting conclusions. It appears that in an isotropic medium with constant elasticity all the wave surfaces are divided into three groups according to whether they have rectilinear polarization in the directions of curvature lines. A sphere and circular cylinder belong to the first group; here polarization is possible along each of two curvature lines. Wave surfaces permitting polarization along one of two curvature lines belong to the second; it applies to all surfaces of revolution, in which polarization may take place along only a meridional plane, but not in a plane normal to it. The third group includes all the remaining surfaces which cannot have rectilinear polarization along any of the curvature lines. In the problem of longitudinal oscillations Umov's method led to the same results which had been obtained previously by Poisson but in another way.

Umov extended conclusions obtained concerning transverse oscillations in an infinite medium to optical phenomena. Taking into account some additional assumptions on properties of a medium which propagates light oscillations (ideal elasticity, low density, etc.) Umov obtains optical equations corresponding to those of Boussinesq. This may be easily proved by transforming Boussinesq differential equations obtained in his work *New Theory of Light Waves* (1868) to curvilinear co-ordinates.

2.

Umov's work *Theory of Thermomechanical Phenomena in Solid Elastic Bodies* was presented as a thesis for master's degree of physico-mathematical sciences.

The public defence of the thesis was very successful. It took place at the Moscow University in 1872 under the chairmanship of the dean of the Physico-mathematical Faculty, the well-known mathematician A. Yu. Davydov. In his summary, Davydov gave high praise to the new work of the young scientist who, by that time, had received an invitation to the Chair of Mathematical Physics in Odessa. His Faculty Professor, Shvedov, who was at that time a Professor of Experimental Physics at the Odessa University, said the following about Umov as a scientist and his thesis devoted to thermomechanical phenomena in solid elastic bodies: "The aim of this work is to link the elasticity theory with the mechanical theory of heat. Not limiting himself to investigation of particular cases of identical temperature, normal pressures and stresses over a whole body, which were considered by Thomson, Clausius and Tseiner, Umov regarded this problem from a general viewpoint—when temperature propagates irregularly over a whole body and the latter suffers various pressures and stresses in different portions. In this case the problem is especially complicated, since temperature at various points in a body varies in time owing to heat conduction, and Umov would have had to deal at once with two theories, i.e. elasticity and heat conduction, based on different principles, if he had not come to the fortunate conclusion of linking these theories by one general principle. For this he used the known principle of conservation of energy. The fact that his equations result in particular cases such as—the second law of thermodynamics, equilibrium equations for solid elastic bodies and the heat-conduction equation—is a confirmation of the value of Umov's investigations."

Umov's thesis for a master's degree is interesting not only from the viewpoint of a pure theoretical investigation but is also of the utmost importance in practice. To calculate elastic stresses due to a non-uniform temperature field in a body is a problem as yet unsolved but of extreme practical importance at the present. Various attempts existing at present are based mainly on the Duamel equations and are of particular solutions only. The statement of the problem made by Umov is more interesting and general; it may therefore yield valuable new results in theory and practice if only it can be followed up by someone capable of bringing the principal ideas of this interesting work closer to present day needs and conditions.

The main content of Umov's work mentioned above may be presented in the following way:

In bodies subjected to simultaneous mechanical and heat effects, an elasticity tensor may be expanded into two tensors—that of molecular elasticity forces and that of heat elasticity forces. Mathematically it may be written as follows:

$$\begin{vmatrix} P_{xx} & P_{xy} & P_{xz} \\ P_{yx} & P_{yy} & P_{yz} \\ P_{zx} & P_{zy} & P_{zz} \end{vmatrix} = \begin{vmatrix} t_{xx} & t_{xy} & t_{xz} \\ t_{yx} & t_{yy} & t_{yz} \\ t_{zx} & t_{zy} & t_{zz} \end{vmatrix} - \begin{vmatrix} p_{xx} & p_{xy} & p_{xz} \\ p_{yx} & p_{yy} & p_{yz} \\ p_{zx} & p_{zy} & p_{zz} \end{vmatrix}$$

or in symbols:

$$P = t - p.$$

Here P, t and p are the tensors of elasticity, heat elasticity forces and molecular elasticity forces, respectively.

All Umov's further arguments are directed to establishing the relation between the elasticity forces mentioned and a deformation tensor:

$$e = \begin{vmatrix} \epsilon_{xx} & \epsilon_{xy} & \epsilon_{xz} \\ \epsilon_{yx} & \epsilon_{yy} & \epsilon_{yz} \\ \epsilon_{zx} & \epsilon_{zy} & \epsilon_{zz} \end{vmatrix}.$$

Regarding any elastic body as a system of mass points connected between themselves by forces of molecular interaction, Umov proves that for an isotropic body in its first approximation there must exist a linear relation between the tensor of molecular elasticity forces and that of a deformation as well as between the tensor of heat elasticity forces and that of deformation, i.e. the following equations must be valid:

$$t_n = \psi(t)[\lambda_1 + K_1\theta + \kappa_1\epsilon_n], \quad t_\tau = \kappa_1\psi(t)\epsilon_\tau;$$

$$p_n = \psi(t)[\lambda + K\theta + \kappa\epsilon_n], \quad p_\tau = \kappa\epsilon_\tau.$$

Here the subscripts n and τ symbolize normal and tangential stresses, respectively.

The function $\psi(t)$ depends in a certain way on the initial state of an elastic body provided that t is the temperature of the centre of gravity of an elementary volume.

Umov gives the following form of this function if the initial pressure, acting on the surface of a body, is designated as P_0:

$$\psi(t) = \frac{P_0 + \lambda}{\lambda_1} + ba_1(t - t_0).$$

Here the coefficient of linear expansion of the bodies is designated as a_1, whilst b is expressed by

$$b = \frac{3K + \kappa}{\lambda_1} - \frac{P_0 + \lambda}{\lambda_1} \cdot \frac{3K_1 + \kappa_1}{\lambda_1}.$$

Thus, we have all the necessary quantities to write down the equations of motion for an elastic body.

The equations derived were applied by Umov to several specific problems, in particular to those of Clausius and Thomson.

3.

Umov's work *Equations of the Motion of Energy in Bodies* resulted from his two other works published in *Mathematical Collections* in 1873 and entitled *The Theory of Interaction at Finite Distances and its Application to the Derivation of Electric and Electrodynamic Laws* and *The Theory of Simple Media*. It was submitted by Umov to the Moscow University as a doctor's thesis.

In this work Umov developed the idea that potential energy cannot appear in one simple medium, it is essential for there to be, at least, two media the second of which, not amenable to direct observation (latent medium), takes up some of the kinetic energy and thus, defines (or determines) our assumption of the existence of potential energy. Umov says: "Potential energy is none other than the kinetic energy of motions of several media of which we are not directly aware." From this viewpoint he formulates the energy conservation law in the following way:

(a) Any change in the value of kinetic energy is caused by its transition from particles of one medium to those of the other media or from some forms of motion to the other;
(b) A certain amount of kinetic energy remains the same for any change of phenomena;
(c) The total kinetic energy is invariable in nature.

Proceeding from this concept, Umov shows by means of some simple assumptions on motion of particles in latent media, how it is possible quantitatively to express basic laws of interaction of electric charges, magnetic poles, electric currents, etc.

Kinetic energy is always bound up with a moving particle, and it is there where a particle is. Hence there arose the concept on the motion of energy. Umov was the first to confirm this concept and to popularize it widely considering that the concept on energy density and the velocity of its motion may be applied to any form of energy.

Umov worked out differential equations for the motion of energy in solids with constant elasticity and in liquids. In various particular cases integration of these equations leads to conclusions of great fundamental importance. Applying his ideas to propagation of waves in an elastic medium, Umov comes to the conclusion that energy is entirely transferred by a wave from one point to another, and he suggests the following simple theorem: "The amount of energy passing through a surface element of a body per unit time is equal to the force of pressure or stress acting on this element multiplied by the velocity of motion of the element." It is not difficult to see that this theorem, as a matter of fact, does not differ from the Maxwell theorem of light pressure. In 1881 the Dutch scientist Grinwis showed that "Umov's law", as he called it, may be successfully applied to collision phenomena of elastic bodies.

Umov's ideas, developed in his doctor's thesis, considerably influenced the further development of energy concepts. Later on, in 1884, Umov's ideas were developed by the English physicist Poynting in application to an electromagnetic field.

Umov's thesis on energy motion in bodies is important not only in the history of physics but has also retained its value as a strictly materialistic work presenting effective methods for developing continuum dynamics and field theory.

Umov was the first to state the problem of creating such a general motion theory, based on the most general properties of material in which there would be no hint of reducing one type of motion to another. This problem is the most pressing one in modern science. It is a pity that it took more than seventy years for attention to be paid to this remarkable attempt of the famous Russian physicist.

Localizing energy in space Umov clearly understood at the same time that it is possible only with simultaneous localization of material.

He says: "With the present development of natural sciences only the idea of successive motion transition from one material particles to another, infinitely close together, can correspond to the idea of gradualness mentioned. This problem was solved in this direction. And properties of the intermediate medium were usually equated to those of media already investigated, but the choice itself of any of the media was never determined on the basis of any general guiding principles. Therefore explanations existing in science for some particular forms of interaction do not bear the stamp of inevitability, which is desirable for explaining natural phenomena."

All the profundity of Umov's thinking is expressed in these words. He approached the investigation of natural phenomena clearly understanding the main theoretical and cognitive viewpoints by which a naturalist should be guided. Umov proposed an equation for energy motion in two forms.

If a continuum does not contain sources, one of these equations for energy motion, from the viewpoint of modern mathematics, may be written as follows:

$$\frac{\partial E}{\partial t} + \operatorname{div} \boldsymbol{\sigma} = 0.$$

Here E is the energy density and $\boldsymbol{\sigma}$ is the Umov vector. This equation corresponds to the following proposition of Umov: "There are always three functions σ_x, σ_y and σ_z possessing the property that the sum of the first derivatives along axes x, y and z involves the decrease in energy density per time unit at a given point of a body."

For the medium with constant elasticity Umov's vector is as follows:

$$\boldsymbol{\sigma} = T_{ik}\,\mathbf{v}.$$

Here T_{ik} is the tensor of elastic stresses and $\mathbf{v}$, the velocity vector of elastic displacements.

For a viscous liquid Umov's vector is given thus:

$$\boldsymbol{\sigma} = \mathbf{v}\frac{\rho v^2}{2} + T_{ik}\,\mathbf{v}.$$

Here T_{ik} is the tensor of elastic and viscous forces.

For an ideal liquid the previous expression may be re-written as follows:

$$\boldsymbol{\sigma} = \mathbf{v}\left(p + \frac{\rho v^2}{2}\right).$$

From the latter it follows that Umov's vector in an ideal liquid is always directed along stream lines and this leads to the following important theorem of hydrodynamics for an ideal liquid: "Bodies immersed in a 'correct' stream of ideal fluid do not suffer frontal drag."

This proposition is not valid for a viscous liquid since there Umov's vector does not coincide with the stream lines.

Umov's ideas, when properly used, may serve as a basis for developing calculation methods of hydrodynamic resistances.

Professor Kessenikh from the Thomsk University gave an interesting example regarding application of Umov's ideas. In the paper entitled "Umov's vector and connected mass"* this scientist demonstrated the fruitfulness of Umov's method when solving the following problem: how the energy of connected mass is localized in an acoustic field and how to single it out from the total energy of an acoustic field. Proceeding from Umov's equation, this problem is fully solved by extremely simple considerations.

4.

The doctor's thesis submitted by Professor E. Leist in 1899 to the Physico-mathematical Faculty of the Moscow University gave rise to three remarkable works on geomagnetism by Umov. The subject of this thesis was formulated as follows: "On the Geographical Distribution of Normal and Anomalous Geomagnetism." The Faculty entrusted Umov to review Leist's work. With his usual thoroughness he not only studied Leist's thesis but stated a number of new problems.

As a result of five-years work carried out by Umov, the science of terrestrial magnetism was enriched by two extremely important works, viz:

(1) Some Experience Gained in the Investigation of Magnetic Specimens of Terrestrial Magnetism;
(2) Construction of a Geometric Specimen of the Gauss Potential as a Method of Determining Laws of Terrestrial Magnetism.

The Gauss theory, or rather, twenty-four dead, empirically determined coefficients of the expansion of terrestrial magnetism potential in series of harmonic functions, was the basis of these works. In due course the deep theoretical significance of these coefficients was forgotten, and the Gauss expansion of terrestrial magnetism potential was reduced to a simple interpolation formula. Instead of searching for the physical meaning of the main expansion coefficients, efforts were directed to calculating higher terms of a potential for obtaining better agreement between the Gauss formula and the actual distribution of terrestrial magnetism. For example, Adams calculated forty-eight coefficients and Fritsche sixty-three, but this did not save the position and the latter acknowledged that the addition of terms in harmonic functions, even of the seventh order, does not give the necessary accuracy.

In Umov's work the Gauss coefficients were represented again and they acquired the necessary physical and geometrical meaning. Comparing geometrical properties of harmonic functions with the known facts of geographical distribution of terrestrial magnetism, Umov came to the conclusion that not all the expansion coefficients of the Gauss potential with respect to harmonic functions have the same value; some of them are united into complete groups and form the so-called main magnetic types; others are only supplementary to the main types, they specify the geographical distribution of terrestrial magnetism given by the coefficients referring to the main magnetic types. Umov reduced all 24 Gauss coefficients to the main four types and thus he transformed the confused pattern of empirical expressions regarding distribution of terrestrial magnetism into a clear and scientifically substantiated picture.

All this resulted from a detailed understanding of geometric properties of harmonic functions.

Harmonic functions are geometrical representations having several axes which allow extremely simple polar equations to be derived for lines of equal potential.

For example, in the case of harmonic equations of the second order such an equation is expressed by the product of the cosines of the angles of a radius-vector at a point on a curve with axes of a harmonic function providing it is constant for one and the same line. At the same time Umov noticed that there exists some relation between the position of these lines, the axes and several straight lines connected with them and

* Vestnik Moskovskogo Universiteta, 1949.

some singularities in magnetic terrestrial properties. For example, the bisector of an angle between the axes of a second order harmonic passes through the middle of a region of the East Asian magnetic anomaly, whilst the borders of this region hardly differ from a certain equipotential line of this harmonic. Besides this Umov discovered the following remarkable circumstance: long term changes in the elements of terrestrial magnetism reduce to a very rapid displacement of some axes of the harmonic function with very slow displacement of the other axes of the same function. Further Umov determined that its geometrical image (representation) bears a definite relation to the axes of rotation of the earth. Umov completed his analysis with a third order harmonic function leaving it to future generations to study higher order harmonics.

Thus Umov's works on terrestrial magnetism were so significant that he undoubtedly ranks with Gauss in this respect.

5.

From 1886 Umov began to be interested in experimental work. Since then, for some years he devoted his attention to problems of diffusion phenomena in aqueous solutions. Umov carried out experimental work with youthful enthusiasm and very often stayed in the laboratory far into the night. His note-books are brimful of plans for experimental work. After experiments on a certain subject he sometimes came to an abrupt halt, collected together the results accumulated into his bag, with the object of returning to it after a while. In the intervening periods Umov usually occupied himself with other experimental work.

In 1887 Umov published his first experimental work, *Solubility Laws of Several Salts.*

In 1888 Umov published an extensive experimental work *Diffusion of an Aqueous Salt Solution.* He used the known Faraday–Thomson method, experimented for four months and found a number of simple integral laws characterizing a diffusion process. One of these laws reads: the ratio of the lengths of two columns of solution bounded by layers of equal density is independent of time.

In 1889–1891 Umov also investigated the diffusion of acid solutions. The work *An Addition to the Hydrodiffusion Law and New Diffusion Meters* resulted from these observations. Here a number of ingenious devices for observing diffusion such as: "the syphon diffusion meter", "diffusion hook", "diffusion aerometer", etc., are described. In this work Umov criticized Fick's hypothesis from fresh standpoints, viz: he points out that during diffusion the addition of solute solvent to the existing solution is accompanied by compression or expansion. Fick's hypothesis does not take this change of volume into account. Introducing an additional term taking into consideration this effect, Umov corrects the Fick equation. Umov's works on diffusion in general represent a model experimental investigation, both in profundity of theoretical analysis, and in the development of new experimental methods.

In 1905 the publication of the work *Chromatic Depolarization Resulting from Light Dispersion* was the first of a series of Umov's investigations on optics. He discovered that if a beam of polarized light rays is directed onto a matt surface, this beam is more or less depolarized with respect to those rays which pass through a body to a maximum degree and, vice versa, the rays which are absorbed to the greatest extent show the greatest polarization. If the incident light is not polarized, then in a dispersed beam those beams which are most of all absorbed by a body appear to be the most polarized. This optical phenomenon should be referred to as Umov's effect. In scientific and educational literature it is given as the following rule: If we direct rays of various colours onto a coloured matt surface, those colours which are diffusively reflected without absorption evince depolarization; and vice versa, those rays which are partially absorbed by a given substance are polarized by it. For instance, red cloth depolarizes red rays and polarizes green ones. Umov took this very phenomenon to be a basis of his method for a spectral analysis of matt surfaces. In 1909 for this he designed the apparatus consisting of a spectroscope with a horizontal axis. The substance being investigated is placed on the table of this spectroscope. Rays passing from a collimator slot are diffusively reflected from the surface investigated; a part of these rays enters a tube which is supplied with a Savart's plate, nicol and the prism giving a spectrum.

If a reflecting surface polarizes light, then in the spectrum observed partial dark lines will be obtained owing to the interference action of the Savart's plate; if, however, the light reflected from the surface investigated does not undergo polarization, then the dark bands mentioned above vanish. If the apparatus is used to observe painted surfaces which have an absorption spectrum the following results will be manifest: in those spectrum positions where absorption did not take place and where, consequently, there was no light polarization, narrowing or even full annihilation of Savart's bands will be observed; on the contrary, in the locations with absorption, intensification and broadening of these bands will be observed. Thus the spectrum of rays reflected from the surface considered is presented in the form of spots or "beads" superimposed on the Savart's lines. There are two reasons for this pattern: polarization of diffusively reflected light, and its attenuation on account of absorption in the given substance. The pattern seen in Umov's spectroscope is characteristic for each substance concerned as is shown by numerous specimens mentioned in his work. By the way, Umov carried out similar observations on chlorophyll as well and obtained two characteristic series of beads in red and orange rays.

Umov's method is very sensitive, therefore Umov expressed the view that the presence of chlorophyll on planets might be determined by this method. Moreover, rocks of our earth are investigated by this method; the pattern obtained being compared with the corresponding one obtained from various points on the moon surface, and thus the mineral composition of the moon surface may be determined.

Umov worked on the development of the optical phenomenon described until his death. After leaving the University as a result of the incident in 1911 at the age of sixty-six, he organized a laboratory on the premises of the Higher Technical College, which was kindly allocated to him by Professor Petrov. Here he investigated absorption of some hundreds of various substances with the help of the apparatus he designed and made by the German firm Füss.

6.

In 1875 during his trip abroad Umov submitted his work *On the Steady Motion of Electricity on Arbitrary Conducting Surfaces* for Kirchhoff's consideration.

Before the work mentioned above, this problem was solved only for several particular cases; Kirchhoff solved it for a plane, Boltzmann for a sphere and a circular cylinder, whereas Umov solved it in its very general form. Umov reduced the problem of distribution of electric currents on any surface to that of the current distribution in a flat plate, representing a conformal transformation of the surface considered on a plane. The problem on such transformation of the surface of an arbitrary kind on a plane was theoretically solved by Gauss. Thus, a very difficult problem which was not solved by such eminent scientists as Boltzmann and Kirchhoff was simply and easily solved by Umov.

7.

Umov's work *Geometrical Significance of Fresnel's Integrals* appeared in 1885. This work once more reveals the endowments of its author as scientist and thinker. Extreme scientific acumen is required to discover new aspects of problems which would normally appear to have been solved long ago. What fresh discovery could come to light in Fresnel's integrals! However, Umov made the discovery; he revealed their geometrical character with extraordinary lucidity. Fresnel's integrals are;

$$A = \int_0^z \cos\frac{\pi}{2}z^2\,dz; \quad B = \int_0^z \sin\frac{\pi}{2}z^2\,dz$$

Umov takes the parabola:

$$\frac{\pi}{2}z^2 = v$$

the axis z being directed vertically and the axis v to the left. In this case we get:

$$A = \int_0^z \cos v\,dz; \quad B = \int_0^z \sin v\,dz.$$

Now imagine a circular cylinder with the radius equal to unity; the axis z will be taken as the axis of this cylinder. Let the upper half of this parabola be wrapped round the cylinder mentioned so that its axis lies round the base circle of the cylinder and the parabola vertex coincides with the intersections between cylinder surface and the axis x. The parabola thus forms a helix.

If we project this helix onto the co-ordinate planes zx and zy, then the abscissae of the points of these projections may be written as follows:

$$x = \cos v, \quad y = \sin v.$$

Now Fresnel's integrals take the following form:

$$A = \int_0^z x \, dz; \quad B = \int_0^z y \, dz.$$

The problem therefore of calculating Fresnel's integrals is reduced to quadratures. Umov gives a method for calculating corrections to the values of integrals A and B calculated by Fresnel himself, presents approximate formulas for determining extremum values of light intensity in a diffraction pattern formed by an edge of an infinite screen; then he describes the design of an integrator to do mechanical calculations of integrals A and B.

The whole work bears an impress of elegance and simplicity.

8.

Two of Umov's works, *Uniform Derivation of Transformations Consistent with Relativity Principle* and *Invariance Conditions of the Wave Equation*, show how vividly he responded to ideas in the field of theoretical physics which were current at that time. The outstanding Russian scientist Joukovski considers these works of Umov's to be the best mathematical interpretation of the relativity principle. This is what Joukovski writes about this work*: "Just as non-Euclidian geometry and geometry of many measurements are based on the invariance of the generalized concept of an element of arc, the relativity principle due to Umov has its mathematical content in invariance of the wave equation in the propagation of light."

In order to solve the problem in question Umov transforms the wave equation for four-dimensional space by introducing the imaginary variable $\tau = cti$ instead of a time co-ordinate, where c is the velocity of light; then he specifies that this equation should remain invariant when passing from co-ordinates x, y, z and τ to x', y', z' and τ'. It appears that this may only be done if the second differential parameters of the functions x', y', z' and τ', expressed through variables x, y, z and τ, are equal to zero.

When x, y and τ are parameters of a Cartesian system of co-ordinates and $z = z' = 0$, invariance of the wave equation demands that x', y' and τ' should be parameters of an isothermal system of curvilinear tri-orthogonal co-ordinates. In particular, x', y' and τ' may be considered to be parameters of the Cartesian system of co-ordinates turned through an imaginary angle φi relative to the axis y. Further, taking $\tan \varphi = v/c$, finally, Umov arrived at the formulas of the Lorentz–Einstein transformation.

In 1913 he devoted considerable effort to the composition of a speech which he was to deliver at the forthcoming first All-Russian Conference of Teachers of Physics, Chemistry and Cosmography. The Conference took place in St. Petersburg. Umov's speech was entitled "Evolution of Physical Science and its Conceptual Significance". In his speech, Umov threw light on the development of the newest atomistic theories in details. The speech was delivered on 29 December, 1913 (Julian calendar), and accepted with great enthusiasm. Whilst preparing his speech, Umov hit upon ideas which can only be adequately understood today.

Modern wave mechanics resulted from searching for a solution to the problem of the interaction between an electromagnetic field and matter (more precisely an electron). It is based on the principle which may be formulated as follows: experimentally absolutely precise determination of two canonically conjugated parameters, characterizing the system of state, is impossible. This determination may be done only to a known approximation, the degree of which is determined by the following inequalities: for the co-ordinate of position and the corresponding impulse

$$-\Delta x \,.\, \Delta p_x \geqslant h,$$

* Collection dedicated to Umov's memory, published by Ledentsov's Society, 1915.

and for particle energy

$$-\Delta E \,.\, \Delta t \geqslant h.$$

In his work *A Possible Interpretation of the Quantum Theory*, published twelve years before basic ideas of wave mechanics appeared, Umov writes the following: "An electromagnetic field with respect to chaotic motions of particles possesses various degrees of sensitivity." In other words: it is impossible to have a quite exact quantitative expression for interaction between an electromagnetic field and the chaotic motion of particles; we may conceive only average values, characterizing the state of a system.

In his opinion no apparatus exists which would have as high a degree of sensitivity for determining any interactions as Maxwell's demon possesses. This is the only apparatus which has infinite sensitivity; any other records a phenomenon with known and finite approximation. If we use an electric field to evaluate interaction between a latent medium and "visible" matter, then the sensitivity of this apparatus must be finite as well. This apparatus possesses some average sensitivity with respect to chaotic motions of molecular systems. The value of this average sensitivity must leave its mark on all the average values which one encounters in the process of getting to know the laws of nature. Umov takes a value equal to $1/h\nu$ as a measure of average sensitivity of an electromagnetic field as an apparatus with the help of which one learns to apprehend the laws of nature. Here ν is the repeatability of natural oscillations of a molecular system, and h the Planck constant.

The considerations expressed show neither more nor less than the following:

$$\frac{\Delta E}{\nu} \geqslant h \quad \text{or} \quad \Delta E T \geqslant h.$$

Using only the Maxwell distribution law, Umov determined a formula for the average energy of the Planck resonator on the basis of the concept mentioned. Thus, one sees how closely Umov approached modern ideas of wave mechanics at that time. As a matter of fact he was the first who, in the history of development of the quantum theory, dared to talk about an approximate character of measurements of parameters determining the state of a system.

9.

From Umov's works under review it is seen that he was a scientist with a wide outlook and whose interests were not channelled into a few favourite scientific problems. He responded to all which was new and fundamental that appeared in the process of scientific progress.

His investigations on heat and mass transfer in elastic and fluid media have placed him amongst the outstanding scientists in the field of molecular physics.

In spite of the generalizing character of his investigations Umov was not a scientist over inclined to abstraction. Being engaged in science he deeply understood its high aims. In his paper *Cultural Role of Physical Sciences* he wrote: "Physical sciences by their content and traditions have raised thought so far above the ordinary levels and have made such an impact on the vital interests of mankind that for them the aphorism 'science for science' has lost its meaning. However specialized the ideas, experiment and measurement, in addition to the measurements of a scientific worker, they still contribute to either world outlook or material progress."

Umov as both scientist and human being becomes distinctly apparent from his statements on various occasions.

He writes: "Nature did not classify the human being as a one-stringed instrument, but as a many-stringed instrument, in a variety of tribes and peoples. It is against natural laws to break the strings of this wonderful instrument, and to destroy the rich image of a human being." And further: "The establishment of wide intercourse between people is a characteristic feature of natural sciences: genius and labour do not want to admit obstacles on this path."

"The extraordinary growth in understanding of world phenomena, and the strength of the spiritual bond between scientific workers are achieved in the absence of fear of an idea, however keen and incisive it may be. A concept which from a narrow point of view figures only as a delusion, in the wider scientific field becomes a mighty instrument for confirming truth or is a source of remarkable generalizations. Only by observing a tolerant attitude to thought on a large scale can the immense riches of life and understanding be revealed."

Umov was a naturalist in the full sense of this word. His philosophical statements, his propaganda for science amongst the wider public were always remarkable for the deep love of nature, its beauty and harmony which was revealed. In all natural phenomena he sought to see and to discover reason and laws. Of course, everyone sees extremely fantastic lines bordering a leaf surface of leaf-bearing trees. It did not occur to anybody except Umov, however, to analyse these lines harmonically. Umov did this and determined a number of highly curious laws of which botanists of that time had no idea. He was interested in everything, nothing was ever dull. Such love for nature and craving for knowledge are peculiar only to selected and inquisitive minds.

Umov was one of the chosen and belonged to the foremost names of those who create the material values in the cultural development of the whole of mankind.